



FIXED POINT THEOREMS IN RANDOM FUZZY METRIC SPACE

Dr.S.Sumithiradevi¹ and J.Jenithsilvia²

¹Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi college,
Affiliated to Bharathidasan University, Trichy, Tamilnadu, India.

²M.Sc.Mathematics, Shrimati Indira Gandhi college,
Affiliated to Bharathidasan University, Trichy, Tamilnadu, India

Abstract:

Fixed point theorems in random fuzzy metric space using three and four mappings which satisfying contractive condition.

Key words:

Fuzzy metric space, Random fuzzy metric space, Weakly compatible.

1.Introduction

The concept of fuzzy set was introduced by Zadeh and fuzzy metric space was developed by Kramosil and Michalek. Sharma, Sharma and Iseki studied contraction type mapping in random fuzzy metric space.

The concept of fuzzy random variable was introduced as an analogous notion to random variable in order to extend statistical analysis to situations when the outcomes of some random experiment are fuzzy sets. But in contrary to the classical statistical methods no unique definition has been established the work of Volker. After that many interesting random fixed point results and several applications have appeared in the literature for example the work of Beg and Shahzad. Gupta, Dhagat, Shrivastava introduced the fuzzy random spaces and proved common fixed point theorem. In this paper, we give fixed point theorems in random fuzzy metric space using three and four self mappings which satisfying contractive condition with uniqueness.



2. Basic Preliminaries

Definition:2.1

A binary operation $*$: $[0,1] \times [0,1] \rightarrow [0,1]$ is a t-norm if it satisfies the following conditions:

- $*$ is commutative and associative
- $*$ is continuous
- $a*1 = a$ for all $a \in [0,1]$
- $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$, and $a,b,c,d \in [0,1]$

Definition:2.2

The triple $(X, M, *)$ is a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, M is a fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions:

- (i) $M(x,y,0) = 0$
- (ii) $M(x,y,t) = 1, \forall t > 0$ if and only if $x=y$
- (iii) $M(x,y,t) = M(y,x,t)$
- (iv) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s), \forall x,y,z \in X$ and $t,s > 0$.
- (v) $M(x,y,.) : [0, \infty) \rightarrow [0,1]$ is left continuous.

Definition:2.3

Let $(X, M, *)$ be a fuzzy metric space. A mapping $f: X \rightarrow X$ is fuzzy contractive if there exists $0 < k < 1$ such that $\frac{1}{M(f(x), f(y), t)} - 1 \leq k \left[\frac{1}{M(x,y,t)} - 1 \right]$

For each $x,y \in X$ and $t > 0$.

Definition:2.4

Let $(X, M, *)$ be a fuzzy metric space. We will say that the sequence $\{x_n\}$ in X is fuzzy contractive if there exists $k \in (0,1)$ such that

$$\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1 \leq k \left[\frac{1}{M(x_n, x_{n+1}, t)} - 1 \right] \text{ for all } t > 0, n \in \mathbb{N}.$$

We recall that a sequence $\{x_n\}$ in a metric space (X, d) is said to be contractive if there exist $0 < k < 1$ such that $d(x_{n+1}, x_{n+2}) \leq k d(x_n, x_{n+1})$ for all $n \in \mathbb{N}$.

Definition:2.5



Throughout this chapter, (Ω, Σ) denotes a measurable space. $\xi : \Omega \rightarrow X$ is a measurable selector. X is any non empty set, $*$ is continuous t- norm, M is a fuzzy set in $X^2 \times [0, \infty)$

Definition:2.6

The 3- tuple $(X, M, \Omega, *)$ is called a Random fuzzy metric space, if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions for all $\xi_x, \xi_y, \xi_z \in X$ and $s, t > 0$

- $M(\xi_x, \xi_y, 0) = 0$
- $M(\xi_x, \xi_y, t) = 1$ for all $t > 0$, if and only if $x = y$
- $M(\xi_x, \xi_y, t) = M(\xi_y, \xi_x, t)$
- $M(\xi_x, \xi_z, t+s) \geq M(\xi_x, \xi_y, t) * M(\xi_y, \xi_z, s)$
- $M(\xi_x, \xi_y, \xi_a) : [0, 1] \rightarrow [0, 1]$ is left continuous.

In follows $(X, M, \Omega, *)$ will denote an random fuzzy metric space. Note that $M(\xi_x, \xi_y, t)$ can be thought of as the degree of nearness between ξ_x and ξ_y with respect to t . We identify $\xi_x = \xi_y$ with $M(\xi_x, \xi_y, t) = 1$ for all $t > 0$ and $M(\xi_x, \xi_y, t) = 0$ with ∞ . In the following example, we know that every metric induces a fuzzy metric.

Example:2.7

Let (X, d) be a metric space.

Define $a * b = ab$ or $a * b = \min \{a, b\}$ and for all $x, y \in X$ and $t > 0$

$$M(\xi_x, \xi_y, t) = \frac{t}{t + d(\xi_x, \xi_y)}$$

Then $(X, M, \Omega, *)$ is a fuzzy metric space. we call this random fuzzy metric M induced by the metric d the standard fuzzy metric.

Definition:2.8

A sequence $\{\xi x_n\}$ in a random fuzzy metric space $(X, M, \Omega, *)$ is called Cauchy if $\lim_{n \rightarrow \infty} M(\xi x_{n+p}, \xi x_n, t) = 1$ for all a in X and $t > 0, p > 0$.

A sequence $\{\xi x_n\}$ in a random fuzzy metric space $(X, M, \Omega, *)$ is said to be Convergent to $x \in X$ $\lim_{n \rightarrow \infty} M(\xi x_n, \xi x, t) = 1$ for all a in X and $t > 0$.

A random fuzzy metric space $(X, M, \Omega, *)$ is said to be complete if every Cauchy sequence in X converges in X .

Definition:2.9



A function M is continuous in fuzzy metric space iff whenever $\xi x_n \rightarrow \xi x, \xi y_n \rightarrow \xi y$ which implies $\lim_{n \rightarrow \infty} M(\xi x_n, \xi y_n, t) \rightarrow M(\xi x, \xi y, t)$

Definition:2.10

Two mapping A and S on fuzzy metric space X are weakly commuting iff

$$M(AS\xi u, SA\xi u, t) \geq M(A\xi u, S\xi u, t)$$

Lemma:2.11

For all $\xi x, \xi y \in X, M(\xi x, \xi y, t)$ is non-decreasing.

Lemma:2.12

Let $\{\xi y_n\}$ be a sequence in a random fuzzy metric space $(X, M, \Omega, *)$ if there exists a number $q \in (0, 1)$ such that $M(\xi y_{n+2}, \xi y_{n+1}, qt) \geq M(\xi y_{n+1}, \xi y_n, t)$ for all $t > 0$ and $n = 1, 2, 3, \dots$ then $\{\xi y_n\}$ is a cauchy sequence in X .

Lemma:2.13

For all $\xi x, \xi y \in X, t > 0$ and for a number $q \in (0, 1), M(\xi x, \xi y, qt) \geq M(\xi x, \xi y, t)$ then $\xi x = \xi y$.

Definition:2.14

A binary operation $*$: $[0, 1] \times [0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t-norm if $\{[0, 1], *\}$ is an abelian topological monoid with unit 1 such that $a_1 * b_1 * c_1 \geq a_2 * b_2 * c_2$ whenever $a_1 \geq a_2, b_1 \geq b_2, c_1 \geq c_2$, for all $a_1, b_1, c_1, a_2, b_2, c_2 \in [0, 1]$.

Preposition:2.15

Let (X, d) be a metric space. The mapping $f: X \rightarrow X$ is a contractive (a contraction) on the metric space (X, d) with contractive constant k if and only if f is fuzzy contractive, with contractive constant k , on the standard fuzzy metric space $(X, M_d, *)$ induced by d .

Preposition:2.16

Let $(X, M, *)$ be a complete fuzzy metric space in which fuzzy contractive sequences are cauchy. Let $T: X \rightarrow X$ be a fuzzy contractive mapping being k the contractive constant. Then T has a unique fixed point.

Preposition:2.17

Let $(X, M_d, *)$ be the standard fuzzy metric space induced by the metric d on X . The sequence $\{x_n\}$ in X is contractive in (X, d) if and only if $\{x_n\}$ is fuzzy contractive in $(X, M_d, *)$.



Proposition 2.25 and 2.27 imply that proposition 2.26 is a generalization of Banach fixed point theorem to fuzzy metric spaces as defined by George and Veeramani.

It is to be noted that all the propositions are true for (Random fuzzy metric).

3. Main Result

Theorem:3.1

Let $(X, \Omega, M, *)$ be a complete Random fuzzy metric space in which fuzzy contractive sequences are Cauchy and A, S and T be mappings from $(X, \Omega, M, *)$ into itself $\xi : \Omega \rightarrow X$ is a measurable selector satisfying the following conditions:

$$A(X) \subseteq S(X) \text{ and } A(X) \subseteq T(X)$$

$$\frac{1}{M(A(\xi x), A(\xi y), t)} - 1 \leq k \left(\frac{1}{Q(\xi x, \xi y, t)} - 1 \right) \text{ with } 0 < k < 1 \text{ and}$$

$$Q(\xi x, \xi y, t) = \varphi[\min\{M(S\xi x, T\xi y, t), M(A\xi x, S\xi x, t), M(A\xi y, T\xi y, t), M(A\xi x, A\xi y, t), M(A\xi x, T\xi y, t)\}]$$

Where $\varphi : (0, 1) \rightarrow (0, 1)$ is continuous function $\varphi(t) = t$. The pairs A, S and T have a unique common fixed point.

Proof:

Let $\xi x_0 \in X$ be an arbitrary point. Since $A(X) \subseteq S(X)$ and $A(X) \subseteq T(X)$, we build a sequence $\{x_n\}$ in X such that $A\xi x_{n-1} = S\xi x_n = T\xi x_n$

Put $x = x_n$ and $y = x_{n+1}$

$$\frac{1}{M(A(\xi x_n), A(\xi x_{n+1}), t)} - 1 \leq k \left(\frac{1}{Q(\xi x_n, \xi x_{n+1}, t)} - 1 \right) \dots(a)$$



$$\begin{aligned}
 Q(\xi x_n, \xi x_{n+1}, t) &= \\
 \varphi \left[\min \{ &M(S\xi x_n, T\xi x_{n+1}, t), M(A\xi x_n, S\xi x_n, t), M(A\xi x_{n+1}, T\xi x_{n+1}, t), M(A\xi x_n, A\xi x_{n+1}, t), \right. \\
 &\left. M(A\xi x_n, T\xi x_{n+1}, t) \} \right] \\
 &= \varphi \left[\min \{ M(A\xi x_{n-1}, A\xi x_n, t), M(A\xi x_n, A\xi x_{n-1}, t), M(A\xi x_{n+1}, A\xi x_n, t), M(A\xi x_n, A\xi x_{n+1}, t), \right. \\
 &\left. M(A\xi x_n, A\xi x_n, t) \} \right] \\
 &= \varphi [\min \{ M(A\xi x_{n-1}, A\xi x_n, t), M(A\xi x_n, A\xi x_{n+1}, t) \}]
 \end{aligned}$$

Case :1

$$M(A\xi x_{n-1}, A\xi x_n, t) \geq M(A\xi x_n, A\xi x_{n+1}, t)$$

$$\text{ie) } Q(\xi x_n, \xi x_{n+1}, t) = \varphi[M(A\xi x_n, A\xi x_{n+1}, t)] = M(A\xi x_n, A\xi x_{n+1}, t) \quad \dots(\text{b})$$

Substitute (b) in (a)

$$\frac{1}{M(A(\xi x_n), A(\xi x_{n+1}), t)} - 1 \leq k \left(\frac{1}{M(A \xi x_n, A \xi x_{n+1}, t)} - 1 \right)$$

which is contradiction.

Case :2

$$M(A\xi x_{n-1}, A\xi x_n, t) < M(A\xi x_n, A\xi x_{n+1}, t)$$

$$\text{ie) } Q(\xi x_n, \xi x_{n+1}, t) = \varphi[M(A \xi x_{n-1}, A \xi x_n, t)] = M(A \xi x_{n-1}, A \xi x_n) \quad \dots(\text{c})$$

Substitute (c) in (a)

$$\frac{1}{M(A(\xi x_n), A(\xi x_{n+1}), t)} - 1 \leq k \left(\frac{1}{M(A \xi x_{n-1}, A \xi x_n, t)} - 1 \right)$$

{ A(ξx_n) } is a fuzzy contractive sequence in (X,Ω,M,*) So { Aξx_n } is Cauchy sequence.

As X is a complete fuzzy metric space, { A ξx_{n-1} } is convergent. So, { A ξx_{n-1} } converges to some point ξz in X.



$\{A\xi x_{n-1}\}, \{S(\xi x_n)\}, \{T(\xi x_n)\}$ converges to ξz .

Put $x = \xi v, y = \xi x_{n-1}$

$$\frac{1}{M(A(\xi v), A(\xi x_{n-1}), t)} - 1 \leq k \left(\frac{1}{Q(\xi v, \xi x_{n-1}, t)} - 1 \right) \quad \dots(d)$$

$$\begin{aligned} Q(\xi v, \xi x_{n-1}, t) &= \\ \varphi \left[\min \{ &M(S\xi v, T\xi x_{n-1}, t), M(A\xi v, S\xi v, t), M(A\xi x_{n-1}, T\xi x_{n-1}, t), M(A\xi v, A\xi x_{n-1}, t), \right. \\ &\left. M(A\xi v, T\xi x_{n-1}, t) \} \right] \\ &= \varphi[\min \{M(S\xi v, \xi z, t), M(A\xi v, S\xi v, t), M(\xi z, \xi z, t), M(A\xi v, \xi z, t), M(A\xi v, \xi z, t)\}] \\ &= \varphi[M(A\xi v, \xi z, t)] = M(A\xi v, \xi z, t) \quad \dots(e) \end{aligned}$$

Substitute (e) in (d)

$$\frac{1}{M(A(\xi v), \xi z, t)} - 1 \leq k \left(\frac{1}{M(A\xi v, \xi z, t)} - 1 \right)$$

Which is contradiction

$$A\xi v = \xi z$$

$$\xi z = A\xi v = S\xi v$$

Similarly, $T\xi v = \xi z$

$$\xi z = A\xi v = T\xi v$$

(A,S) and (A,T) are weakly compatible

$$AS\xi v = SA\xi v \text{ which implies } A\xi z = S\xi z$$

$$AT\xi v = TA\xi v \text{ which implies } A\xi z = T\xi z$$

Hence ξz is common fixed point of A,S and T.

Uniqueness:

Now, $\xi v \neq \xi z$

$$\begin{aligned} Q(\xi v, \xi z, t) &= \varphi[\min \{ (M(S \xi v, T \xi z, t), M(A \xi v, S \xi v, t), M(A \xi z, T \xi z, t), M(A \xi v, A \xi z, t), M(A \xi v, T \xi z, t)) \} \\ &= \varphi[\min \{ (M(\xi v, \xi z, t), M(\xi v, \xi v, t), M(\xi z, \xi z, t), M(\xi v, \xi z, t), M(\xi v, \xi z, t)) \} \\ &= M(\xi v, \xi z, t) \end{aligned}$$



$$\frac{1}{M(\xi_v, \xi_z, t)} - 1 \leq k \left(\frac{1}{M(\xi_v, \xi_z, t)} - 1 \right)$$

which is contradiction

Hence $\xi_v = \xi_z$. A, S and T is unique common fixed point.

Theorem:3.2

Let $(X, \Omega, M, *)$ be a complete Random fuzzy metric space in which fuzzy contractive sequences are Cauchy and A, B be mappings from $(X, \Omega, M, *)$ into itself

$$A(X) \subseteq S(X) \text{ and } B(X) \subseteq T(X)$$

$\xi : \Omega \rightarrow X$ is a measurable selector and $a(\xi) = a > 0$ satisfying the following conditions:

$$\frac{1}{M(A(\xi_x), B(\xi_y), t)} - 1 \leq k \left(\frac{1}{Q(\xi_x, \xi_y, t)} - 1 \right) \text{ with } 0 < k < 1 \text{ and } M(x, y, t) = 1$$

$$Q(\xi_x, \xi_y, t) = \phi \left[\max \{ M(S\xi_x, T\xi_y, t) * M(A\xi_x, S\xi_x, t), M(A\xi_x, B\xi_y, t), M(S\xi_x, B\xi_y, t) * M(B\xi_x, T\xi_y, t) \} \right]$$

where $\phi : (0, 1) \rightarrow (0, 1)$ is continuous function $\phi(t) = t$. The pairs A, S, B, and T have a unique common fixed point.

Proof:

Let $\xi x_0 \in X$ be an arbitrary point. $A(X) \subseteq S(X)$ and $B(X) \subseteq T(X)$ we build a sequence $\{x_n\}$ in X such that $\xi x_n = S\xi x_n = A\xi x_{n-1} = B\xi x_{n+1} = T\xi x_{n+2}$

Put $x = \xi x_n$ and $y = \xi x_{n+1}$

$$\frac{1}{M(A(\xi x_n), B(\xi x_{n+1}), t)} - 1 \leq k \left(\frac{1}{Q(\xi x_n, \xi x_{n+1}, t)} - 1 \right) \quad \dots(a)$$



$$Q(\xi x_n, \xi x_{n+1}, t) =$$

$$\phi [\max\{ M(S\xi x_n, T\xi x_{n+1}, t) * M(A\xi x_n, S\xi x_n, t), M(A\xi x_n, B \xi x_{n+1}, t), M(S\xi x_n, B\xi x_{n+1}, t) * M(B\xi x_n, T\xi x_{n+1}, t)\}]$$

$$= \phi \left[\max\{M(A\xi x_{n-1}, B \xi x_n, t)M(A\xi x_n, A\xi x_{n-1}, t), M(A\xi x_n, B \xi x_{n+1}, t), M(A\xi x_{n-1}, B \xi x_{n+1}, t) * M(B \xi x_{n+1}, B \xi x_{n-1}, t)\} \right]$$

$$= \phi[\max\{M(A\xi x_{n-1}, B \xi x_n, t), M(A\xi x_n, B \xi x_{n+1}, t), M(A\xi x_n, B \xi x_{n+1}, t)\}]$$

Case : 1

$$M(A \xi x_{n-1}, B \xi x_n, t) \geq M(A \xi x_n, B \xi x_{n+1}, t)$$

$$\text{ie) } Q(\xi x_n, \xi x_{n+1}, t) = \phi[M(A \xi x_{n-1}, B \xi x_n, t)] = M(A \xi x_{n-1}, B \xi x_n, t) \dots(\text{b})$$

Substitute (b) in (a)

$$\frac{1}{M(A(\xi x_n), B(\xi x_{n+1}), t)} - 1 \leq k \left(\frac{1}{M(A(\xi x_n), B(\xi x_{n+1}), t)} - 1 \right)$$

which is contradiction.

Case :2

$$M(A \xi x_n, B \xi x_{n+1}, t) < M(A \xi x_n, B \xi x_{n+1}, t)$$

$$\text{ie) } Q(\xi x_n, \xi x_{n+1}, t) = \phi[M(A \xi x_n, B \xi x_{n+1}, t)] = M(A \xi x_n, B \xi x_{n+1}, t) \dots(\text{c})$$

substitute (c) in (a)

$$\frac{1}{M(A(\xi x_n), B(\xi x_{n+1}), t)} - 1 \leq k \left(\frac{1}{M(A \xi x_n, B \xi x_{n+1}, t)} - 1 \right)$$



$\{A(\xi x_n)\}$ is a fuzzy contractive sequence in $(X, \Omega, M, *)$ So $\{A\xi x_n\}$ is Cauchy sequence.

As X is a complete fuzzy metric space, $\{A\xi x_{n-1}\}$ is convergent. So, $\{A\xi x_{n-1}\}$ converges to some point ξz in X .

$\{A\xi x_{n-1}\}, \{S\xi x_n\}, \{T\xi x_n\}, \{B(\xi x_{n-1})\}$ converges to ξz .

Put $x = \xi v, y = \xi x_{n-1}$

$$\frac{1}{M(A(\xi v), B(\xi x_{n-1}), t)} - 1 \leq k \left(\frac{1}{Q(\xi v, \xi x_{n-1}, t)} - 1 \right) \quad \dots(d)$$

$$Q(\xi v, \xi x_{n-1}, t) =$$

$$\phi [\max\{ M(S\xi v, T\xi x_{n-1}, t) * M(A\xi v, S\xi v, t), M(A\xi v, B\xi x_{n-1}, t), M(S\xi v, B\xi x_{n-1}, t) * M(B\xi x_{n-1}, T\xi x_{n-1}, t) \}]$$

$$= \phi [\max\{ M(A\xi v, \xi z, t) * M(A\xi v, A\xi v, t), M(A\xi v, \xi z, t), M(A\xi v, \xi z, t) * M(\xi z, \xi z, t) \}]$$

$$= \phi [\max\{ M(A\xi v, \xi z, t) * (1), M(A\xi v, \xi z, t), M(A\xi v, \xi z, t) * (1) \}]$$

$$= \phi [\max\{ M(A\xi v, \xi z, t) \}]$$

$$= M(A\xi v, \xi z, t) \quad \dots(e)$$

Substitute (e) in (d)

$$\frac{1}{M(A(\xi v), (\xi z), t)} - 1 \leq k \left(\frac{1}{M(A\xi v, \xi z, t)} - 1 \right)$$

which is contradiction

$$A\xi v = \xi z$$

$$\xi z = A\xi v = S\xi v$$

put $x = \xi x_n$ and $y = \xi v$



$$\frac{1}{M(A\xi x_n, B\xi v, t)} - 1 \leq k \left(\frac{1}{Q(\xi x_n, \xi v, t)} - 1 \right) \quad \dots(k)$$

$$Q(\xi x_n, \xi v, t) =$$

$$\left[\phi \left[\max \{ M(S\xi x_n, T\xi v, t) * M(A\xi x_n, S\xi x_n, t), \right. \right. \\ \left. \left. M(A\xi x_n, B\xi v, t) M(S\xi x_n, B\xi v, t) * M(B\xi v, T\xi v, t) \} \right] \right]$$

$$= \phi \left[\max \{ M(\xi z, B\xi v, t) * M(\xi z, \xi z, t), M(\xi z, B\xi v, t), M(\xi z, B\xi v) * M(B\xi v, B\xi v, t) \} \right]$$

$$= \phi \left[\max \{ M(B\xi v, \xi z, t) * (1) M(B\xi v, \xi z, t), M(B\xi v, \xi z, t) * (1) \} \right]$$

$$= \phi \left[M(B\xi v, \xi z, t) \right]$$

$$= M(B\xi v, \xi z, \xi a, t) \quad \dots(l)$$

Substitute (l) in (k)

$$\frac{1}{M(B(\xi v), (\xi z), \xi a, t)} - 1 \leq k \left(\frac{1}{M(B\xi v, \xi z, \xi a, t)} - 1 \right)$$

which is contradiction

$$B\xi v = \xi z$$

$$\xi z = B\xi v = T\xi v$$

(A,S) and (B,T) are weakly compatible

$$AS\xi v = SA\xi v \text{ which implies } A\xi z = S\xi z$$

$$BT\xi v = TB\xi v \text{ which implies } B\xi z = T\xi z$$

Hence ξz is common fixed point of A,S,B and T.

Uniqueness:



Now, $\xi_v \neq \xi_z$

$$\frac{1}{M(A\xi_v, B\xi_z, t)} - 1 \leq k \left(\frac{1}{Q(\xi_v, \xi_z, t)} - 1 \right)$$

$$Q(\xi_v, \xi_z, t) = \phi \left[\max\{ M(S\xi_v, T\xi_z, t), M(A\xi_v, S\xi_v, t), M(A\xi_v, B\xi_z, t), M(S\xi_v, B\xi_z, t), M(B\xi_z, T\xi_z, t) \} \right]$$

$$= \phi [\max\{ M(\xi_z, \xi_v, t) * M(\xi_z, \xi_z, t), M(\xi_z, \xi_v, t), M(\xi_z, \xi_v, t) * M(\xi_v, \xi_v, t) \}]$$

$$= \phi [\max\{ M(\xi_z, \xi_v, t) * (1), M(\xi_z, \xi_v, t), M(\xi_z, \xi_v, t) * (1) \}]$$

$$= M(\xi_z, \xi_v, t)$$

$$\frac{1}{M(\xi_z, \xi_v, t)} - 1 \leq k \left(\frac{1}{M(\xi_z, \xi_v, t)} - 1 \right)$$

which is contradiction.

Hence $(\xi_v) = (\xi_z)$ A, S, B and T have a unique fixed point.

CONCLUSION:

In this paper, we have established new common fixed point theorems for three and four self-mappings in the setting of complete random fuzzy metric spaces under suitable contractive conditions. By constructing appropriate iterative sequences and employing the properties of fuzzy contractive sequences, completeness, measurable selectors, and weak compatibility, we proved the existence and uniqueness of a common fixed point for the given mappings. Hence, the present study contributes to the ongoing development of fixed point theory in generalized metric structures that incorporate both fuzziness and randomness. The obtained results may find applications in nonlinear analysis, stochastic models, decision theory, and mathematical models involving uncertainty. Future investigations may focus on relaxing compatibility conditions,



considering multivalued mappings, or extending the results to other generalized spaces such as probabilistic fuzzy metric spaces and intuitionistic fuzzy metric spaces.

References:

- [1] Dubey. R.P, Bhardwaj. R. Neeta Tiwari, Ankur Tiwari (2013) Fixed point theorems in Random fuzzy metric space Through Rational Expression, Computer Engineering and Intelligent Systems Vol.4, No.9,pp:14-32.
- [2] George,A. and Veermani.P.(1994): On some results in fuzzy metric spaces, Fuzzy sets and Systems.
- [3] Grabiec,M.(1988) Fixed points in fuzzy metric space, Fuzzy sets and systems, Vol.27,pp:385-389.
- [4] Gregori,V and Sepena,A(2002) On fixed point theorems in fuzzy metric spaces, Fuzzy sets and systems 125-245.
- [5] Kramosil.J and Michalek.J (1975) Fuzzy metric and statistical metric spaces, Kymbernetica, 11,330.
- [6] Schweizer.B and Sklar.A, Probablistic metric spaces,North Holland, Amsterdam, 1983.
- [7] Sumithiradevi.S,Muraliraj A Fixed point theorems for Occasionally weakly compatible in Fuzzy metric space (IAS,R) American International Journal of Research in Science Technology,Engineering and Mathematics,pp:53-56(2019).
- [8] Sushil Sharma , On Fuzzy Metric Space, Southeast Asian Bulletin of Mathematics(2002) 26: 133-145.
- [9] Volker, Kratsckhmer(1998), A unified approach to fuzzy – random –variables, Seminar notes in Statistik and Oonometrie Fachbereich Wirtschaftswissenschaft,Universitat des Saarlandes Saarbrucken, Germany, pp:1-17.



[10] Zadeh,L.A(1965)Fuzzy sets , Inform. And Control, Vol.8, pp:338-353.